Math 3794, Practice Problems

Quadratic Polynomials, Fall 2020

Problem 1. Does there exist a quadratic polynomial $p(x)$ such that two of its coefficients are integers and

$$p\left(\frac{1}{2020}\right) = \frac{1}{2021}, \quad p\left(\frac{1}{2021}\right) = \frac{1}{2020}?$$

Problem 2. The monic quadratic polynomial $p(x)$ has exactly one real root and the equation

$$p(2x-3) + p(3x+1) = 0$$

also has only one real root. Find the polynomial $p(x)$.

Problem 3. Given the quadratic polynomials $p_1(x)$, $p_2(x)$, $\ldots$, $p_{100}(x)$ with the same coefficients of x^2 and x but with different constant terms. Each of the polynomials has two real roots. For each polynomial $p_i(x)$ one root is denoted by x_i. What values can the sum below have:

$$p_1(x_{100}) + p_2(x_1) + p_3(x_2) + \cdots + p_{100}(x_{99})?$$

Problem 4. Prove that for any positive integer n the polynomial

$$(x+1)^{2n+1} + x^{n+2}$$

is divisible by $x^2 + x + 1$.

Problem 5. If z and t are the roots of the polynomial $x^2 + 2x + 4 = 0$, compute the value of

$$(z+t)^7 - z^7 - t^7.$$

Problem 6. Let a be a real nonzero number. Prove that for the roots z and t of the polynomial

$$x^2 + ax - \frac{1}{2a^2},$$

we have that

$$z^4 + t^4 \geq 2 + \sqrt{2}.$$