LINEAR ALGEBRA PRACTICE SET

1. Compute the determinant

$$D = \begin{vmatrix} 1+a_1 & a_2 & a_3 & ... & a_n \\ a_1 & 1+a_2 & a_3 & ... & a_n \\ ... & ... & ... & ... & ... \\ a_1 & a_2 & a_3 & ... & 1+a_n \end{vmatrix}.$$

2. Compute the following determinant (a, b are arbitrary)

$$D_{2n} = \begin{vmatrix} a & 0 & 0 & ... & 0 & b \\ 0 & a & 0 & ... & b & 0 \\ ... & ... & ... & ... & ... & ... \\ b & 0 & 0 & ... & 0 & a \end{vmatrix},$$

where the size of the matrix is $2n \times 2n$.

3. Let A be a $n \times n$ matrix, where n is odd. Prove that $\det (A - A^t) = 0$.

4. Let A be an $n \times n$ matrix such that $A^3 = A + I_n$. Prove that $A + I_n$ is invertible.

5. Let A be a 2×2 matrix so that $tr\,(A) = tr\,(A^2) = 0$. Prove that $\det (A) = 0$.

6. Let A and B be 2×2 matrices with $\det A = \det B = 1$. Prove that

$$tr\,(AB) - tr\,(A)\,tr\,(B) + tr\,(AB^{-1}) = 0.$$

7. Let A, B be $n \times n$ matrices with real entries so that $AB = BA$. Prove that

$$\det (A^2 + B^2) \geq 0.$$

8. Calculate

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}^n.$$