Math 3794, Practice Problems

Polynomials, Spring 2020

Problem 1. Solve the equation

$$x^2 + \frac{81x^2}{(9+x)^2} = 40.$$

Problem 2. Solve the system of equations for x, y, and z

$$\begin{aligned} x + y + z &= a \\ x^2 + y^2 + z^2 &= b^2 \\ xy &= z^2 \end{aligned}$$

where a and b are parameters.

Problem 3. Prove that there is no polynomial p with integer coefficients such that $p(7) = 5$ and $p(15) = 9$.

Problem 4. Prove that for any positive integer n the polynomial

$$(x+1)^{2n+1} + x^{n+2}$$

is divisible by $x^2 + x + 1$.

Problem 5. Consider polynomials T_n and U_n given by the formulas

$$T_n(\cos\theta) = \cos n\theta, \quad U_n(\cos\theta) = \frac{\sin(n+1)\theta}{\sin\theta}, \quad n = 0, 1, 2, \ldots.$$

Show that

$$T_{n+1}(x) = xT_n(x) - (1-x^2)U_{n-1}(x).$$

Problem 6. Prove that for any nonzero a and b, the roots x_1, x_2, and x_3 of the polynomial $ax^3 - ax^2 + bx + b$ satisfy the relation

$$(x_1 + x_2 + x_3)\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}\right) = -1.$$