

# Math 3794 Problem Seminar, Homework 6

Due Tuesday, November 19th

**Problem 1 (Difficulty: 2).** Let  $x \in \mathbb{R}$ . Prove that

$$\sin(\cos x) < \cos(\sin x).$$

**Problem 2 (Difficulty: 2).** Let  $a$ ,  $b$ , and  $c$  be positive real numbers. Show that

$$\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b} \geq \frac{3}{2}.$$

**Problem 3 (Difficulty: 2).** Let  $n$  be a positive integer and let  $a_1, a_2, \dots, a_n$  be  $n$  positive real numbers. Prove that

$$\frac{a_1^2}{a_2} + \frac{a_2^2}{a_3} + \dots + \frac{a_{n-1}^2}{a_n} + \frac{a_n^2}{a_1} \geq a_1 + a_2 + \dots + a_{n-1} + a_n.$$

**Problem 4 (Difficulty: 1).** Let  $x \in \mathbb{R}$ . Prove that

$$\sin^6 x + \cos^6 x \geq \frac{1}{4}.$$

**Problem 5 (Difficulty: 2).** For a positive integer number  $n$ , define

$$z_n = \sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}},$$

where the square root is taken  $n$  times. Show that

$$\frac{2 - z_n}{2 - z_{n-1}} > \frac{1}{4}.$$