Math 3794 Problem Seminar Fall 2019 Combinatorics

- 1. How many 4 letter words of the 26 letter English alphabet are possible when:
- a.) (Difficulty: 1) Repetition of letters is allowed?
- b.) (Difficulty: 1) Repetition of letters is *not* allowed?
- c.) (Difficulty: 1) Repetition of letters is allowed, but not consecutively?

2. Of your textbooks, you have 4 that are math, 3 physics, 3 English, and 2 history. You would like to order them on a shelf. How many ways can you order then if:

- (a) (Difficulty: 1) the math books have to all come first, English second, physics third, and history fourth?
- (b) (Difficulty: 2) you only need to keep all the books of the same subjects together?

3. You are ordering frozen yogurt from a place with 16 varieties of frozen yogurt and 36 toppings. How many different combinations are possible using:

- (a) (Difficulty: 1) A single frozen yogurt flavor and one topping?
- (b) (Difficulty: 1) A single frozen yogurt flavor and two different toppings?
- (c) (Difficulty: 2) At least one frozen yogurt flavor and any number of toppings? Your answer should not have a summation.

4. Consider walks on the integers starting from 0 and going either +1 or -1 at each step.

- (a) (Difficulty: 1) How many different walks of length 12 are possible? Two walks are different if they differ at any point.
- (b) (Difficulty: 2) How many different walks of length 12 end at 2?
- (c) (Difficulty: 3) How many different walks of length 12 go to 4 before ending at 2?

5. (Difficulty: 5) Let N be a positive integer. How many ways are there to order the integers $\{1, ..., 3N+1\}$ (as say $x_1, ..., x_{3N+1}$) so that $\sum_{i=1}^{k} x_i$ is not divisible by 3 for any $k \in \{1, ..., 3N+1\}$? Your answer should be in closed form (no summations) but may include factorials.