Math 3794 Problem Seminar Fall 2019 Probability

1. Two factories make light bulbs for a market. Bulbs from factor X work for over 5000 hours in 98% of cases whereas bulbs from factor Y work for over 5000 hours in 96% of cases. Factor X is known to supply 55% of all bulbs.

a.) (Difficulty: 1) What is the probability that a purchased bulb will work for longer than 5000 hours?

b.) (Difficulty: 1) Given that a light bulb does not work more than 5000 hours, what is the probability it came from factor Y?

2. Let $X, Y \in [0, 2]$ be chosen uniformly at random.

(a) (Difficulty: 1) What is the probability that $X + Y \leq 1$?

(b) (Difficulty: 2) What is the probability that $XY \leq 1$?

3. All die in this problem are fair and 6-sided.

(a) (Difficulty: 1) You roll a dice and receive money equal to the number on top of the dice. What is your expected payout?

(b) (Difficulty: 1) You roll two dice and receive money equal to the sum of the numbers on top of the dice. What is your expected payout?

(c) (Difficulty: 2) You roll one dice and, after seeing the number on top, can choose to either stop or reroll exactly one more time. You receive money equal to the number on top of the dice at the very end. What is your optimal strategy and its expected payout?

4. We randomly place n balls into m bins.

(a) (Difficulty: 1) What is the probability that any randomly chosen bin is occupied by a ball?

(b) (Difficulty: 2) What is the expected number of occupied bins? (Your answer should a fairly simple term and not contain summations or factorials)

5. (Difficulty: 5) You have coins $C_1, C_2, \cdots, C_n$. For each k, coin C_k is biased so that, when tossed, it has probability $1/(2k+1)$ of falling heads. If the n coins are tossed, what is the probability that the number of heads is odd? Express the answer as a rational function of n.