

Math 3974 Problem Seminar Homework 2

Due October 8, 2019

Problem 1.1. (Difficulty:2) Consider five arbitrary points inside an equilateral triangle of side length 2. Show that at least 2 of the points are within 1 unit distance from each other.

Problem 1.2. (Difficulty:1) Given 12 different 2-digit numbers, show that one can choose two of them so that their difference is a two-digit number with identical first and second digit.

Problem 1.3. (Difficulty:2) Fifteen children together gathered 100 nuts. Prove that some pair of children gathered the same number of nuts.

Problem 1.4. (Difficulty:3) Consider any five points P_1, \dots, P_5 in the interior of a square S of side length 1. Show that one can find two of the points at distance at most $1/\sqrt{2}$ apart. Show that this is the best possible.

Problem 1.5. (Difficulty:6) Suppose that 5 points lie on a sphere. Prove that there exists a closed semi-sphere (half a sphere including boundary circle), which contains 4 of the points.

Problem 1.6. (Difficulty:6) If each square of the 3-by-7 rectangle below is colored either black or white, then the board must contain a rectangle (consisting of at least four squares) whose corner squares are either all white or all black.

