LINEAR ALGEBRA SET 1, DUE APRIL 29

1(2 pts). Compute

$$D=\begin{vmatrix} 1+a_1 & a_2 & a_3 & ... & a_n \\ a_1 & 1+a_2 & a_3 & ... & a_n \\ ... & ... & ... & ... & ... \\ a_1 & a_2 & a_3 & ... & 1+a_n \end{vmatrix}.$$

2 (3 pts). Compute the following determinant (a, b are arbitrary)

$$D_{2n}=\begin{vmatrix} a & 0 & 0 & ... & 0 & b \\ 0 & a & 0 & ... & b & 0 \\ ... & ... & ... & ... & ... & ... \\ b & 0 & 0 & ... & 0 & a \end{vmatrix},$$

where the size of the matrix is $2n \times 2n$.

3 (4 pts). Let A be an $n \times n$ matrix whose entry on the i-th row and j-th column is

$$\frac{1}{\min(i,j)},$$

for all $i, j = 1, .., n$. Compute $\det A$.

4 (1 pt). Let A be a $n \times n$ matrix, where n is odd. Prove that $\det(A - A^t) = 0$.

5 (2 pts). Let A be an $n \times n$ matrix such that $A^3 = A + I_n$. Prove that $A + I_n$ is invertible.

6 (2 pt). Let A be an $n \times n$ matrix whose entries are odd integers. Show that $\det A$ is divisible by 2^{n-1}.

7 (2 pt). Let A be a 2×2 matrix so that $tr\,(A) = tr\left(A^2\right) = 0$. Prove that $\det(A) = 0$.

8 (1 pt). Let A and B be 2×2 matrices with $\det A = \det B = 1$. Prove that

$$tr\,(AB) - tr\,(A)\,tr\,(B) + tr\left(AB^{-1}\right) = 0.$$