1 (2 pts). Compute
\[ D = \begin{vmatrix}
1 + a_1 & a_2 & a_3 & \ldots & a_n \\
a_1 & 1 + a_2 & a_3 & \ldots & a_n \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a_1 & a_2 & a_3 & \ldots & 1 + a_n \\
\end{vmatrix}. \]

2 (3 pts). Compute the following determinant \((a, b \text{ are arbitrary})\)
\[
D_{2n} = \begin{vmatrix}
a & 0 & 0 & \ldots & 0 & b \\
0 & a & 0 & \ldots & b & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
b & 0 & 0 & \ldots & 0 & a \\
\end{vmatrix},
\]
where the size of the matrix is \(2n \times 2n\).

3 (4 pts). Let \(A\) be an \(n \times n\) matrix whose entry on the \(i\)-th row and \(j\)-th column is
\[
\frac{1}{\min(i, j)},
\]
for all \(i, j = 1, \ldots, n\). Compute \(\det A\).

4 (1 pt). Let \(A\) be a \(n \times n\) matrix, where \(n\) is odd. Prove that \(\det (A - A^t) = 0\).

5 (2 pts). Let \(A\) be an \(n \times n\) matrix such that \(A^3 = A + I_n\). Prove that \(A + I_n\) is invertible.

6 (2 pt). Let \(A\) be an \(n \times n\) matrix whose entries are odd integers. Show that \(\det A\) is divisible by \(2^{n-1}\).

7 (2 pt). Let \(A\) be a \(2 \times 2\) matrix so that \(\text{tr} (A) = \text{tr} (A^2) = 0\). Prove that \(\det (A) = 0\).

8 (1 pt). Let \(A\) and \(B\) be \(2 \times 2\) matrices with \(\det A = \det B = 1\). Prove that
\[
\text{tr} (AB) - \text{tr} (A) \text{tr} (B) + \text{tr} (AB^{-1}) = 0.
\]