Problem 3.1. (Difficulty:1) If $x, y, z, w > 0$, then prove 
\[(x^3y + y^3z + z^3w + w^3x)(x^2y^2 + z^2w^2) \geq 8x^2y^2z^2w^2.\]

Problem 3.2. (Difficulty:1) If $a, b, c > 0$, prove that 
\[a^7 + b^7 + c^7 \geq a^4b^2c + b^4c^2a + c^4a^2b.\]

Problem 3.3. (Difficulty:2) If $a, b, c > 0$, prove that 
\[a^6b + b^6c + c^6a \geq a^4b^2c + b^4c^2a + c^4a^2b.\]

Problem 3.4. (Difficulty:2) If $a, b, c, d > 0$ are given so that $abcd = 1$, prove that 
\[a + b + c + d \leq a^2 + b^2 + c^2 + d^2.\]

Problem 3.5. (Difficulty:3) If $x, y, z > 0$, and $x + y + z = 1$, find the minimal value of 
\[\frac{1}{x} + \frac{1}{y} + \frac{1}{z}.\]

Problem 3.6. (Difficulty:2) For $a, b, c \geq 0$, prove that 
\[\sqrt[3]{9(a + b + c)} \geq \sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}.\]

Problem 3.7. (Difficulty:3) If $a, b, c$ are the sides of a triangle, prove that 
\[\frac{a}{b + c - a} + \frac{b}{c + a - b} + \frac{c}{a + b - c} \geq 3.\]

Problem 3.8. (Difficulty:3) Find the positive solutions of the system of equations 
\[x_1 + \frac{1}{x_2} = 4, \quad x_2 + \frac{1}{x_3} = 1, \quad \ldots, \quad x_{99} + \frac{1}{x_{100}} = 4, \quad x_{100} + \frac{1}{x_1} = 1.\]

Problem 3.9. (Difficulty:3) Find the maximal value of $f(x) = \sin^4 x + \cos^4 x$ for $x \in \mathbb{R}$.

Problem 3.10. (Difficulty:2) Which is larger, $\log_2 3$ or $\log_3 5$?

Problem 3.11. (Difficulty:4) Prove that, for $n = 2, 3, 4, \ldots$
\[n! < \left(\frac{n + 1}{2}\right)^n.\]