Probabilities Homework
Due Monday, February 11

Problem 1. The Sicherman dice are a pair of 6-sided dice oddly numbered 1, 2, 2, 3, 3, 4 and 1, 3, 4, 5, 6, 8 respectively. Show that you can play Catan with these without changing the odds of each outcome. Ignore the initial roll for determining who starts the game.

Problem 2. In the Monty Hall problem, how about if there were 100 doors, you choose 1, then Monty opens 98 bogus doors. What is the probability that the prize is behind the "other" door now?

Problem 3 (The birthday problem)**.** In an arbitrary group of 30 people, none of which is born on Feb 29, what is the probability that at least two of them have the same birthday? Here by birthday we mean the day and month only.

Problem 4. If you toss a fair coin n times, what is the probability that no two consecutive tosses yield heads? (Hint: The probability involves a famous sequence.)

Problem 5. Two players each toss n fair coins. What is the probability that they both get the same number of heads? Your answer should be a simple expression.

Problem 6 (Three points on a circle)**.** Suppose you pick 3 points at random on the unit circle. What is the probability that these points all lie on a semi-circle?
(**Hint**: Use symmetry reasoning to reduce to the case where one of the three points is fixed, say at the "north pole, and the other two points are random points along the circle. Now consider the arcs formed by the breaking up the circle at these points.)

Problem 7 (B1, Putnam 1989)**.** A dart, thrown at random, hits a square target. Assuming that any two parts of the target of equal area are equally likely to be hit, find the probability that the point hit is nearer to the center than to any edge. Simplify the answer.

Problem 8 (2002-B1)**.** Shanille O'Keal shoots free throws on a basketball court. She hits the first and misses the second, and thereafter the probability that she hits the next shot is equal to the proportion of shots she has hit so far. What is the probability she hits exactly 50 of her first 100 shots?