MATH 3974 PROBLEM SEMINAR HOMEWORK 5, DUE NOVEMBER 6

1 (2 pts). For any $x, y, z \in \mathbb{R}$, prove that

$$2^{x^2} + 2^{y^2} + 2^{z^2} \geq 2^{xy} + 2^{yz} + 2^{xz}.$$

2 (2 pts). If $a_1 + a_2 + ... + a_n = n$, prove that

$$a_1^4 + a_2^4 + ... + a_n^4 \geq n.$$

3 (2 pts). If $x_0 > x_1 > ... > x_n$, prove that

$$x_0 + \frac{1}{x_0 - x_1} + ... + \frac{1}{x_{n-1} - x_n} \geq x_n + 2n.$$

4 (2 pts). Prove that for any three positive numbers a_1, a_2, a_3

$$\frac{a_1^2 + a_2^2 + a_3^2}{a_1^3 + a_2^3 + a_3^3} \geq \frac{a_1^3 + a_2^3 + a_3^3}{a_1^4 + a_2^4 + a_3^4}.$$

5 (2 pts). Prove that for any real number x,

$$2^x + 3^x - 4^x + 6^x - 9^x \leq 1.$$

6 (3 pts). Show that for all a_1,...,$a_n > 0$ we have

$$\left(\sum_{k=1}^{n} (a_k)^3\right)^2 \leq \left(\sum_{k=1}^{n} (a_k)^2\right)^3.$$

7 (4 pts). Prove that

$$\sqrt{\frac{x}{y+z}} + \sqrt{\frac{y}{x+z}} + \sqrt{\frac{z}{x+y}} > 2$$

for all $x, y, z > 0$.

8 (5 pts). Prove that

$$\frac{a}{b+c} + \frac{b}{c+d} + \frac{c}{d+a} + \frac{d}{a+b} \geq 2$$

for all $a, b, c, d > 0$.

9 (4 pts). Prove that if $x_1, .., x_n \geq 1$ then

$$\sum_{k=1}^{n} \frac{1}{1 + x_k} \geq \frac{n}{1 + \sqrt[n]{x_1 ... x_k}}.$$