GRE prep questions in Analysis

1. Let $f_n(x) = \frac{x^n}{1+x^n}$, $x \in [0,1]$. Which of the following statements hold

    (A) f_n converges pointwise to a function $f : [0,1] \to \mathbb{R}$.

    (B) f_n converges uniforly to a function $f : [0,1] \to \mathbb{R}$

    (C) $\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 \lim_{n\to\infty} f_n(x)dx$.

2. For which x the does the series

$$\sum_{n\in\mathbb{N}} \frac{n!x^{2n}}{n^n(1+x^{2n})}$$

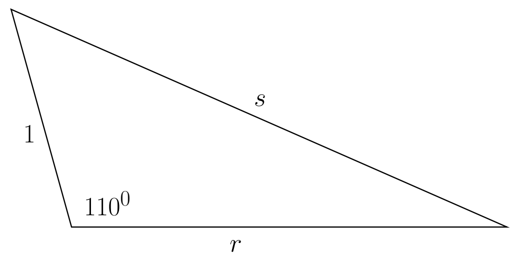
converge?

(A) $\{0\}$

(B) $\mathbb{R}$

(C) $(-1,1)$

(D) $[-1,1]$



3. In the figure above we let r and s increase while keeping one sided fixed with length 1 and the obtuse angle fixed at 110 degrees. Then

$$\lim_{r,s\to\infty} s - r$$

(A) $= 0$,

(B) $\in (0, 1)$,

(C) $= 1$,

(D) $\in (1, \infty)$,

(E) $= \infty$.

4. Let $f : (-1, 4) \to \mathbb{R}$ be a continuously differentiable function such that $f(3) = 5$ and $f'(x) \geq -1$ for all $x \in (1, 4)$. What is the greatest possible value of $f(0)$?

   (A) 3

   (B) 4

   (C) 5

   (D) 8

   (E) 11

5. Which of the following equations has the greatest number of real solutions?

(A) $x^3 = 10 - x$

(B) $x^2 + 5x - 7 = x + 8$

(C) $7x + 5 = 1 - 3x$

(D) $e^x = x$

(E) $\sec x = e^{-x^2}$

6. Find the limit

$$\lim_{z \to 0} \frac{\bar{z}^2}{z^2}, \; z \in \mathbb{C}.$$

(A) 0

(B) 1

(C) i

(D) ∞

(E) The limit does not exist.

7. Let $S \subset \mathbb{R}$. Which of the following statemets is necessarily true?

(A) For all $t, s \in S$ there exists a continuous function $f : [0,1] \to S$ such that $f(0) = s$ and $f(1) = t$.

(B) For each $x \notin S$, there exists an open set $U \subset \mathbb{R}$ such that $u \in U$ and $U \cap S = \emptyset$.

(C) $\{x \in S :$ there exists an open set V such that $x \in V \subset S\}$ is an open subset of $\mathbb{R}$.

(D) $\{x \notin S :$ there exists an open set W such that $x \in W$and $W \cap S = \emptyset\}$ is a closed set.

(E) S is the intersection of all closed subsets of $\mathbb{R}$ that contain S.

8. How many positive solutions does the equation

$$\cos(97x) = x$$

have?

(A) 1

(B) 15

(C) 31

(D) 49

(E) 0

9. Let f, g real functions such that $g(x) = \int_0^x f(y)(y-x)\,dy$. If g is three times continuously differentiable how many times continuously differentiable is f?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

10. Let f, g be twice differentiable functions on $\mathbb{R}$ such that $f'(x) > g'(x)$ for all $x > 0$. Which of the following does it hold for $x > 0$

(A) $f(x) > g(x)$

(B) $f''(x) > g''(x)$

(C) $f(x) - f(0) > g(x) - g(0)$

(D) $f'(x) - f'(0) > g'(x) - g'(0)$

(E) $f''(x) - f''(0) > g''(x) - g''(0)$

11. How many continuous functions $f : [-1, 1] \to \mathbb{R}$ do they exist such that $f(x)^2 = x^2$ for all $x \in [-1, 1]$?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

12. Suppose that f is twice differentiable on $\mathbb{R}$ and that $f(0), f'(0), f''(0) < 0$. Suppose also that f'' has the following properties

(i) It is increasing on $[0, \infty)$.

(ii) It has a unique zero at $[0, \infty)$.

(iii) It is unbounded on the interval $[0, \infty)$.

Which of the above three properties hold also for f?

(A) (i) only.

(B) (ii) only.

(C) (iii) only.

(D) (ii) and (iii) only.

(E) $(i), (ii)$ and (iii).