Math 3974 Problem Seminar Homework 1

Due February 6, 2017

Problem 1.1. (Difficulty:1) [Putnam 1978-A1] Let A be any set of 20 distinct integers chosen from the arithmetic progression $1, 4, 7, \ldots, 100$. Prove that there must be two distinct integers in A whose sum is 104.

Problem 1.2. (Difficulty:1) Show that among any $n+1$ numbers one can find 2 numbers so that their difference is divisible by n.

Problem 1.3. (Difficulty:2) There are five points inside an equilateral triangle of side length 2. Show that at least two of the points are within 1 unit distance from each other.

Problem 1.4. (Difficulty:1) Given 12 different 2-digit numbers, show that one can choose two of them so that their difference is a two-digit number with identical first and second digit.

Problem 1.5. (Difficulty:2) Fifteen children together gathered 100 nuts. Prove that some pair of children gathered the same number of nuts.

Problem 1.6. (Difficulty:3) Consider any five points $P_1, \ldots, P_5$ in the interior of a square S of side length 1. Show that one can find two of the points at distance at most $1/\sqrt{2}$ apart. Show that this is the best possible.

Problem 1.7. (a) (Difficulty:2) Show that, choosing 6 numbers from $\{1, 2, \ldots, 10\}$, there is at least a pair of distinct numbers (a, b) such that a divides b.

(b) (Difficulty:6) Show that, choosing $n+1$ numbers from $\{1, \ldots, 2n\}$, there is at least a pair of distinct numbers (a, b) such that a divides b.

Problem 1.8. (Difficulty:6) Suppose that 5 points lie on a sphere. Prove that there exists a closed semi-sphere (half a sphere including boundary), which contains 4 of the points.

Problem 1.9. (Difficulty:6) If each square of a 3-by-7 chessboard is colored either black or white, then the board must contain a rectangle consisting of at least four squares whose corner squares are either all white or all black.